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EFFECT OF SURFACE EMISSIVITY ON HEAT TRANSFER BY SIMULTANEOUS CONDUCTION AND RADIATION†

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Abstract—This paper is an extension of a previous investigation of the authors' [1] and considers the effect of surface emissivity on temperature distribution and heat transfer in a thermal radiation absorbing and emitting media. Iterative-type solutions of a non-linear integral equation, which governs the temperature field, are obtained for a wide range of parameters. Temperature distributions are given in a graphical form and heat-transfer results are tabulated.

NOMENCLATURE

E, emissive power;
E', irradiation—radiant energy incident on a surface;
$E_n(\tau)$, exponential integral function defined as:
$$E_n(\tau) = \int_0^1 \mu^{n-2} \exp(-\tau/\mu)\, d\mu = \int_0^\infty \exp(-\tau\mu)\mu^{-n}\, d\mu;$$
G, function defined by equation (2);
k, thermal conductivity;
N, dimensionless parameter, $k\kappa/4\sigma T^{*3}$;
n, index of refraction;
q'', total heat flux (conduction + radiation);
q_c'', heat flux by conduction;
q_r'', heat flux by radiation;
R, radiosity—radiant energy leaving a surface;
T, absolute temperature;
T^*, arbitrary temperature;
y, position co-ordinate;
β, dimensionless radiosity, $R/\sigma T^{*4}$;
ϵ, emissivity of the surface;
Θ, dimensionless temperature, T/T^*;
κ, absorption coefficient;
σ, Stefan–Boltzmann constant;
τ, optical depth of the medium defined as $\int_0^y \kappa(y)\, dy$;
τ', dummy integration variable;
τ_0, optical thickness of the medium defined by $\int_0^h \kappa(y)\, dy$.

Subscripts
1 refers to the lower plate;
2 refers to the upper plate.

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INTRODUCTION

IN A recent paper [1] the authors formulated the problem of heat transfer by simultaneous conduction and radiation in an absorbing medium and reported results for the ideal case when the surfaces enclosing the medium are black. This system was essentially two infinite, parallel, isothermal plates separated by an isotropic and homogeneous gray absorbing and emitting medium. It is therefore of interest to study a more realistic situation and certainly a more important engineering case in which the surfaces are gray and are diffuse reflectors. The results of extended calculations which were made to include the effect of surface emissivity on the steady-state temperature distribution and on heat transfer are reported below for the same physical system as in [1].

Since writing the original paper in June of 1960 [1], several new papers related to this topic have come to the authors' attention: Goulard and Goulard [2] studied a one-dimensional problem of simultaneous conduction and radiation. They considered the effects of non-black walls and presented some results for an optically thin ($\tau_0 \ll 1$) gas. Numerical solutions of exact formulations of radiant-heat-transfer problems in an absorbing and heat-generating gas enclosed between two black surfaces were given by Usiskin and Sparrow [3]. Edwards [4], on the other hand, studied radiant heat transfer in an infinite, non-gray, parallel plane enclosure containing an isothermal carbon dioxide–nitrogen gas mixture. He surveyed the existing band data and showed the approximations used and details involved in applying the band approximations to engineering heat-transfer problems. Konakov [5], using a diffusion concept, studied pure radiation and simultaneous conduction and radiation in an absorbing medium between two infinite parallel plates, an infinitely long cylindrical annulus and a spherical annulus.

METHOD OF CALCULATION

The formulation of the problem is given in [1] and will not be repeated here. For convenience the non-linear integral equation giving the normalized temperature distribution, $\Theta = T/T^*$, is given below:

$$\Theta(\tau) = G(\tau) + \frac{1}{2N}\int_0^{\tau_0} n^2(\tau')\,\{-E_3(|\tau-\tau'|) + E_3(\tau') + \tau/\tau_0\,[E_3(\tau_0-\tau') - E_3(\tau')]\}\,\Theta^4(\tau')\,\mathrm{d}\tau', \tag{1}$$

where

$$\left.\begin{aligned} G(\tau) = \frac{1}{2N}\Big\{ & 2N\left[\Theta_1 + \frac{\tau}{\tau_0}(\Theta_2-\Theta_1)\right] \\ & + \beta_1\left[\frac{1}{3}\left(1-\frac{\tau}{\tau_0}\right) + \frac{\tau}{\tau_0}E_4(\tau_0) - E_4(\tau)\right] \\ & + \beta_2\left[\frac{1}{3}\frac{\tau}{\tau_0} + \left(1-\frac{\tau}{\tau_0}\right)E_4(\tau_0) - E_4(\tau_0-\tau)\right]\Big\}. \end{aligned}\right\} \tag{2}$$

The normalized radiosities are given by

$$\beta_1 = \epsilon_1\,\Theta_1^4 + 2(1-\epsilon_1)\,[\beta_2 E_3(\tau_0) + \textstyle\int_0^{\tau_0} n^2(\tau')\,E_2(\tau')\,\Theta^4(\tau')\,\mathrm{d}\tau'] \tag{3}$$

and

$$\beta_2 = \epsilon_2\,\Theta_2^4 + 2(1-\epsilon_2)\,[\beta_1 E_3(\tau_0) + \textstyle\int_0^{\tau_0} n^2(\tau')\,E_2(\tau_0-\tau')\,\Theta^4(\tau')\,\mathrm{d}\tau']. \tag{4}$$

at surfaces $\tau = 0$ and $\tau = \tau_0$, respectively. The first terms in equations (3) and (4) represent emission from the surfaces and the second terms in these equations account for the amount of energy which is incident on the surface and the fraction of this energy which is reflected.

For black surfaces, the function $G(\tau)$ is independent of emissivity and the temperature distribution in the medium for given values of Θ_1 and Θ_2. However, when the surfaces are not black β_1, β_2 and $G(\tau)$ depend on ϵ and the temperature distribution.

The method of successive approximation which was used to solve equation (1) was as follows: A function $\Theta_j(\tau)$ was assumed and inserted into the right-hand side of equations (3) and (4), and the indicated integration was performed. This produced two algebraic equations in two unknowns $\beta_{1,j}$ and $\beta_{2,j}$ [see equations (3) and (4)]. These two equations were then solved for dimensionless radiosities. These radiosities were then used to evaluate G_j. An iteration which was similar to that described in [1] was then followed.

Since the system considered is in steady state, the heat-transfer rate is constant and is the sum of conductive and radiative fluxes. The total heat-transfer rate can be written in dimensionless form as (see equations (23) and (25) in [1]):

$$\left.\begin{aligned} \frac{q''}{\sigma T^{*4}} = 2\Big\{ & \frac{2N}{\tau_0}(\Theta_2-\Theta_1) + \beta_1 \\ & \left[(1-\epsilon_2)\,E_3(\tau_0) + \frac{1}{\tau_0}E_4(\tau_0) - \frac{1}{3\tau_0}\right] \\ & + \beta_2\left[\frac{1}{3\tau_0} - \tfrac{1}{2} - \frac{1}{\tau_0}E_4(\tau_0)\right] + \tfrac{1}{2}\,\epsilon_2\,\Theta_2^4 \\ & + \int_0^{\tau_0} n^2(\tau')\left[(1-\epsilon_2)\,E_2(\tau_0-\tau') + \frac{1}{\tau_0}E_3(\tau_0-\tau) - \frac{1}{\tau_0}E_3(\tau')\right] \Theta^4(\tau')\,\mathrm{d}\tau'\Big\}. \end{aligned}\right\} \tag{5}$$

The left-hand side of this equation is the ratio of the total heat flux to the emissive power of a black body at a temperature T^*. Because the equation governing the temperature distribution is non-linear, equation (5) is the most general expression for dimensionless heat-transfer rate that the authors could find. If only heat conduction was present the heat flux would be given by the first term on the right-hand side of equation (5). The second, third, fourth and fifth terms represent heat transfer by thermal radiation.

DISCUSSION OF RESULTS

Because the problem is non-linear, it is impossible to find a definition of dimensionless temperature that would eliminate the necessity to specify Θ_1 and Θ_2 for a given problem. It also was not possible to cover the complete range of parameters τ_0, N, ϵ_1, ϵ_2, Θ_1, Θ_2 and n. Thus, the parameters were chosen in such a fashion as to shed most light on the problem. To simplify matters, it was assumed that the index of refraction was 1 and that the emissivities of the surfaces were the same, i.e. $\epsilon_1 = \epsilon_2$. This postulate was made in order to reduce the number of parameters entering the problem; no additional difficulty, other than that involved in computation time, would have been introduced if these assumptions were not made. For the sake of completeness, the results for the case $N = 0$, i.e. for pure radiation, are included in this note. The results for this case were obtained in a more general form, in terms of a dimensionless black-body emissive power, in another paper [6].

Before proceeding to discuss the temperature distribution, it is of interest to consider the effect of emissivity on the radiosities. The discussion of the results can then be followed more readily. For small values of τ_0 and $\epsilon < 1$, the radiosity at the surface 1 [see equation (3)] is influenced largely by the radiosity of the second surface, β_2, because $E_3(0) = \frac{1}{2}$, and the effect of radiation from the medium, third term in equation (3), is small. On the other hand, for large values of τ_0, the effect of β_2 on β_1 is very small, in this case, $E_3(10) = 3\cdot5488 \times 10^{-6}$; but the radiation from the medium is more significant. It was found that β_1 increases with the decrease in ϵ, but the contrary is true for β_2; as $\tau_0 \to 0$, $\beta_1 \to \beta_2$.

If the radiant heat flux is essentially constant, or is a small fraction of the total heat flux, the temperature gradient will be practically constant across the medium. This is the case of large N and/or small τ_0. The effect of emissivity is therefore most pronounced for larger values of τ_0 and small values of the parameter N; hence, in the interest of brevity the results will be given only for these cases. For example, in the case $\tau_0 = 0\cdot1$ and $N = 0\cdot01$, the temperature profiles for $\epsilon = 0\cdot1$ differ only by a fraction of 1 per cent from those at $\epsilon = 1\cdot0$.

The effect of emissivity on Θ for $N = 0\cdot01$ and optical thicknesses $\tau_0 = 1\cdot0$ and $\tau_0 = 10\cdot0$ are shown in Figs. 1 and 2, respectively. The temperature profile for the case $\epsilon = 0\cdot9$ is very close to that of $\epsilon = 1\cdot0$; therefore, separate curves were not drawn. It can be seen that the temperature gradients at both surfaces increase with the decrease in ϵ. The effect of emissivity on the temperature distribution at the hot wall is more pronounced for $\tau_0 = 1\cdot0$ than for $\tau_0 = 10\cdot0$. One other fact which is not readily apparent from the curves should be noted:

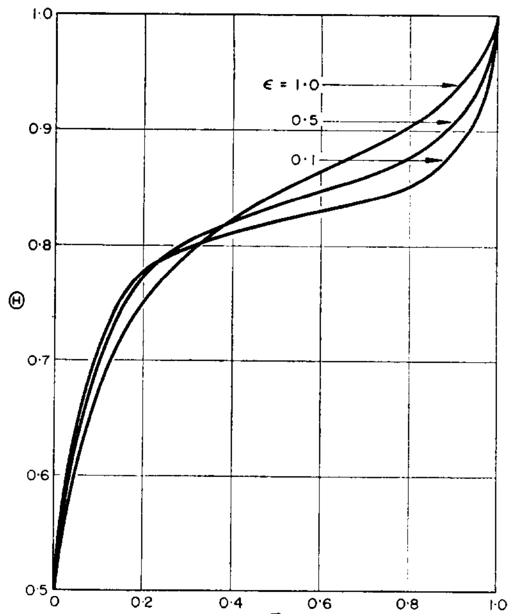

FIG. 1. Variation of temperature with optical depth for $N = 0\cdot01$ and $\tau_0 = 1$.

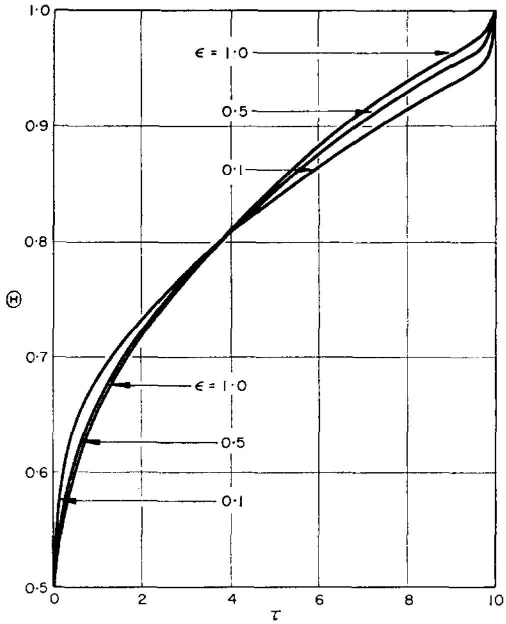

FIG. 2. Variation of temperature with optical depth for $N = 0{\cdot}01$ and $\tau_0 = 10$.

The radiosities β_1 and β_2 are practically independent of each other for this latter case and therefore the temperature profile for a given value of ϵ at a wall is nearly constant with the emissivity of the other surface.

Figure 3 shows the temperature profiles for the case of pure radiation ($N = 0$). For a given ϵ the tendency towards flatness increases with the decreasing τ_0. In an enclosure where the optical thickness is small, there is negligible absorption of energy, no matter where in the enclosure the energy was emitted. Since any point in the medium may transfer heat directly to the wall without recourse to other points, there is no reason for the temperature gradient to exist. The results for $\tau_0 = 0{\cdot}1$ closely approximate this situation. The effect of emissivity for a given value of τ_0 is to further flatten the temperature profile. For large optical thickness, the energy is transferred from one element of the medium to its neighbor, and then finally to the wall. The thermal resistance associated with this type of transport process gives rise to a temperature gradient, as evidenced by inspection of Fig. 3. The transfer of energy between adjacent elements is quite similar to molecular heat conduction. Note the interesting result that the temperature level at the cool surface increases with the decrease in τ_0, but the reverse is true at the hot surface. In passing, it is interesting to note that the results of Usiskin and Sparrow [3] and those of this study for black surfaces, even though given in terms of different parameters, are, within the accuracy of numerical calculations, the same.

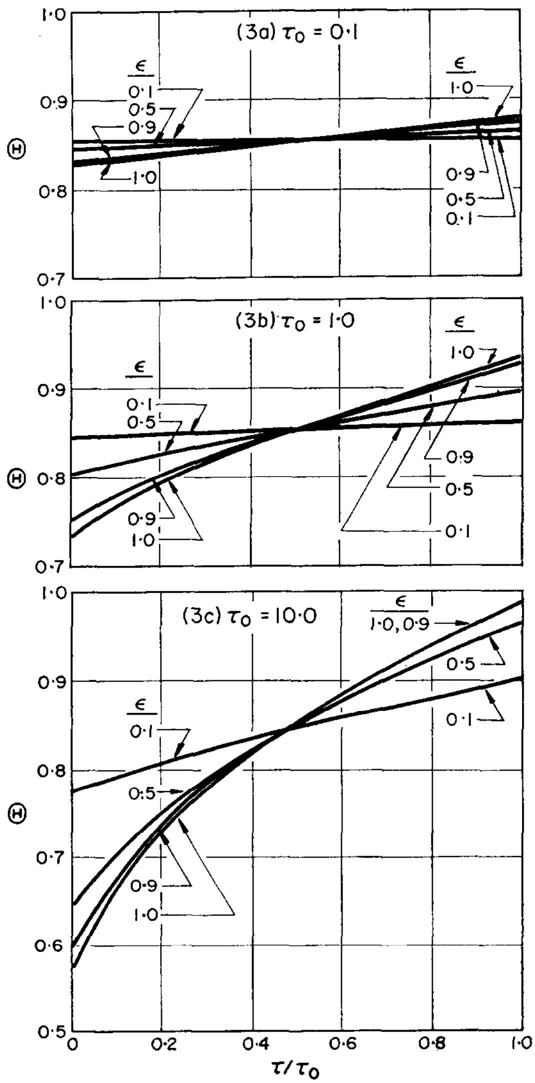

FIG. 3. Temperature profiles for $\Theta_1 = 0{\cdot}5$ and $\Theta_2 = 1{\cdot}0$ when $N = 0$.

The temperature profiles obtained by Goulard and Goulard [2] at the cool wall do not agree in trend with those of this note. For a case $N < 0{\cdot}01$ and $\tau_0 \ll 1{\cdot}0$ they show a decrease in temperature gradient with the decrease in emissivity. This is probably due to the fact that in their case the hot surface was completely transparent and the thermal conductivity and the absorption coefficients varied with temperature.

Since the system considered here is in a steady state, the total energy transport, conduction plus radiation, across the medium is constant. To insure this, it is necessary for variations in the energy flux by radiation to be compensated by opposite variations in the conductive energy flux. Thus, from the temperature profiles in Figs. 1 and 2, we can conclude that in the vicinity of the surfaces the energy transfer by conduction is a larger fraction of the total energy flux than in the regions further removed from the walls. The temperature gradients in the neighborhood of the walls, as was already noted, increase with the decrease in emissivity, and therefore the energy flux by conduction in these regions becomes a larger fraction of the total energy flux with higher reflectivity walls.

Referring to equation (5), we note that the thermal conductivity is independent of temperature, and the heat transfer by conduction, first term in equation (5), does not depend on the emissivity. The conductive flux depends only on the temperature gradient which exists as if the medium were not radiating.

The results of heat-transfer calculations are given in Table 1. Both the dimensionless heat flux, $q''/\sigma T^{*4}$, and the ratio of the heat flux by conduction to the total heat flux, q_c''/q'', are given. This last ratio indicates most clearly the

Table 1. Results of heat-transfer calculations

ϵ/N	0		0·01		0·1		1·0		10	
	$q''/\sigma T^{*4}$	q_c''/q''	$q''/\sigma T^{*4}$	q_c''/q''	$q''/\sigma T^{*4}$	q_c''/q''	$q''/\sigma T^{*4}$	q_c''/q''	$q''/\sigma T^{*4}$	q_c''/q''
(a) $\tau_0 = 0{\cdot}1, \Theta_1 = 0{\cdot}5, \Theta_2 = 1{\cdot}0$										
1·0	0·859	0	1·074	0·186	2·880	0·694	20·88	0·9579	200·88	0·9956
0·9	0·713	0	0·928	0·216	2·723	0·734	20·73	0·9648	200·73	0·9964
0·5	0·309	0	0·524	0·389	2·332	0·858	20·33	0·9838	200·34	0·9983
0·1	0·049	0	0·267	0·750	2·078	0·962	20·08	0·9960	200·08	0·9996
(b) $\tau_0 = 1{\cdot}0, \Theta_1 = 0{\cdot}5, \Theta_2 = 1{\cdot}0$										
1·0	0·518	0	0·596	0·0335	0·798	0·252	2·600	0·769	20·60	0·9709
0·9	0·462	0	0·523	0·0382	0·743	0·269	2·555	0·783	20·55	0·9732
0·5	0·248	0	0·338	0·0592	0·457	0·437	2·397	0·834	20·39	0·9809
0·1	0·047	0	0·156	0·1284	0·393	0·509	2·245	0·891	20·25	0·9977
(c) $\tau_0 = 1{\cdot}0, \Theta_1 = 0{\cdot}1, \Theta_2 = 1{\cdot}0$										
1·0	0·556	0	0·658	0·0547	0·991	0·363	4·218	0·853	36·60	0·9836
0·9	0·495	0	0·581	0·0620	0·968	0·372	4·171	0·863	36·57	0·9845
0·5	0·274	0	0·390	0·0923	0·742	0·485	3·994	0·901	36·37	0·9898
0·1	0·051	0	0·222	0·1620	0·591	0·609	3·752	0·960	36·22	0·9939
(d) $\tau_0 = 10{\cdot}0, \Theta_1 = 0{\cdot}5, \Theta_2 = 1{\cdot}0$										
1·0	0·102	0	0·114	0·0175	0·131	0·153	0·315	0·635	2·114	0·9461
0·9	0·100	0	0·111	0·0180	0·130	0·154	0·314	0·637	2·113	0·9465
0·5	0·084	0	0·104	0·0192	0·122	0·164	0·307	0·651	2·110	0·9479
0·1	0·036	0	0·090	0·0222	0·115	0·174	0·297	0·673	2·107	0·9492

contribution of conduction and radiation to the total heat flux.

The results of Table 1(d) with respect to the influence of emissivity on heat transfer by radiation ($N = 0$) are in agreement with those of [5]. The equation for radiant-energy flux, valid for $\tau_0 > 2$, derived by Konakov is

$$q_r''/\sigma T^{*4} = \frac{\Theta_2^4 - \Theta_1^4}{1/\epsilon_1 + 1/\epsilon_2 - 1 + (\tau_0 - 2)}. \quad (6)$$

This equation is identical in form, except for the factor $(\tau_0 - 2)$, to an equation derived by Eckert and Drake [7] for a non-radiating medium. The deviations between the predictions of the approximate equation (6) and the results based on the solution of the integral equation (1) are not more than 3 per cent for the case when $\tau_0 = 10$.

For simultaneous conduction and radiation, Konakov deduced an equation, equation (72) of [5], for the total heat flux which in the notation of this paper becomes

$$q''/\sigma T^{*4} = q_c''/\sigma T^{*4} + q_r''/\sigma T^{*4} = 2N(\Theta_2 - \Theta_1) + \frac{\Theta_2^4 - \Theta_1^4}{1/\epsilon_1 + 1/\epsilon_2 - 1}. \quad (7)$$

Equation (7) is valid only for $\tau_0 < 2$. Note that the first term on the right-hand side of this equation, expressing the heat transfer by conduction, is smaller by a factor of $2/\tau_0$ than the first term on the right-hand side of equation (5). For $\tau_0 = 0{\cdot}1$ the conductive energy flux predicted by equation (7) is 20 times smaller than that of this note. The greatest deviation for radiant heat flux between the predictions of the simple equation (7) and the results of Table 1(d) occur for $\epsilon_1 = \epsilon_2 = 1$. It is not surprising that Konakov's simplified analysis fails to predict correctly the conductive and radiative energy fluxes. The diffusion-type approximation is not expected to be valid for small optical thicknesses.

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Zusammenfassung—Eine frühere Untersuchung der Autoren [1] ist hier fortgesetzt mit einer Betrachtung des Einflusses des Oberflächenemissionsvermögens auf die Temperaturverteilung und den Wärmeübergang in einem Medium, das thermische Strahlung absorbiert und emittiert. Für einen grossen Bereich von Parametern ergaben sich Iterativlösungen für die das Temperaturfeld kennzeichnende nichtlineare Integralgleichung. Die Temperaturverteilungen sind grafisch angegeben, die Ergebnisse der Wärmeübertragung tabelliert.

Аннотация—Статья является развитием предыдущего исследования авторов [1] и рассматривает влияние степени черноты поверхности на теплоперенос и распределение температуры в средах, поглощающих и испускающих тепловое излучение. Для нелинейного интегрального уравнения, описывающего температурное поле, получены решения разностного типа, пригодные в широком диапазоне изменения параметров. Распределения температуры представлены графически, а характеристики теплопереноса —в виде таблиц.